

Chapter notes: 15 Further integration

Overview

This chapter focuses on various methods for integration, particularly parts, substitution and trigonometric identities. It requires approximately eight hours of teaching time.

Introductory problem

This problem provides an opportunity for students to prove a result which should already be familiar, but which has perhaps not yet been explained. Students who are very perceptive may point out that in the derivation of the derivative of $\sin x$ we effectively assumed this result, so unfortunately it is a circular proof! The worked solution is given at the end of the chapter, page 462; the idea being that students should be able to answer the question using the methods covered in the chapter.

15A Reversing standard derivatives, p442

This is a shortcut version of integration by substitution. It is not required in the syllabus, but it is extremely useful. Question 6 highlights the fact that the expressions we get when integrating may differ by a constant.

15B Integration by substitution, p446

As well as describing the mechanics of integration by substitution, we have tried to emphasise when to choose this method. Students frequently use integration by parts in situations where integration by substitution is more suitable.

The syllabus states that non-standard substitutions will be provided. An example of this might be the trigonometric substitution in Worked example 19.5.

Hints for grade 7 questions:

8. (b) Substitute $u = \cos x$.
9. Substitute $u = x^2 - 3x + 3$, and simplify algebraically before integrating.

15C Kinematics, p453

It is particularly important that students remember the constant of integration when dealing with kinematics questions.

Hints for the grade 7 questions:

8. Find v and then set $\frac{dv}{dt} = 0$.

15D Volumes of revolution, p457

The formula for surfaces of revolution around the x -axis, as mentioned in the ‘Research explorer’ box on page 460, is:

$$2\pi \int y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

It might be useful to get students to derive this.